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Supporting Information

Thickness-dependent superconductivity in MoRe films studied by terahertz spectroscopy

E. S. Zhukova,[1] B. M. Nekrasov,[1] L. S. Kadyrov,[1] A. V. Melentev,[1] A. S. Shaimardanov,[1,2] A. G. Shishkin,[1,2,3] A. A. Golubov,[4] M. Yu. Kupriyanov,[2,5] B. P. Gorshunov,[1] and V. S. Stolyarov[1,2,3]

[1] *Moscow Institute of Physics and Technology (National Research University), 141700 Dolgoprudny, Russia*
[2] *National University of Science and Technology (MISIS), 119049 Moscow, Russia*
[3] *Dukhov Research Institute of Automatics (VNIIA), 127055 Moscow, Russia*
[4] *Faculty of Science and Technology and MESA+ Institute for Nanotechnology, University of Twente, 7500 AE Enschede, Netherlands*
[5] *Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991 Moscow, Russia*

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Theory reminder

Complex transmission coefficient for the two-layered system can be evaluated using the following equation [1]:

$$T^*_{1234} = \frac{T_{12}T_{23}T_{34}e^{i(\delta_2+\delta_3)}}{1 + R_{23}R_{34}e^{2i\delta_3} + R_{12}R_{23}e^{2i\delta_2} + R_{12}R_{34}e^{2i(\delta_2+\delta_3)}}, \tag{1}$$

where T_{pq} and R_{pq} are Fresnel transmission and refraction coefficients, respectively, at the boundary between layers with indexes p and q. $\delta_p = \frac{2\pi d_p}{\lambda}(n_p + ik_p)$, for $p = 2, 3$, with d_p being the thickness of the layer p; n_p and k_p are refraction index and extinction coefficient, respectively, of the layer p. Namely, these coefficients can be written in exponential form $T_{pq} = t_{pq}e^{i\varphi^T_{pq}}$ and $R_{pq} = r_{pq}e^{i\varphi^R_{pq}}$ and expanded through refraction index and extinction coefficient of the adjacent layers:

$$t^2_{pq} = \frac{4(n_p^2 + k_p^2)}{(k_p + k_q)^2 + (n_p + n_q)^2}, \tag{2}$$

$$r^2_{pq} = \frac{(n_p - n_q)^2 + (k_p - k_q)^2}{(k_p + k_q)^2 + (n_p + n_q)^2}, \tag{3}$$

$$\varphi^T_{pq} = \arctan\left(\frac{k_p n_q - k_q n_p}{n_p^2 + k_p^2 + n_p n_q + k_p k_q}\right), \tag{4}$$

$$\varphi^R_{pq} = \arctan\left(\frac{2(k_p n_q - k_q n_p)}{n_p^2 + k_p^2 - n_q^2 - k_q^2}\right). \tag{5}$$

Experimental data for the $Mo_{0.6}Re_{0.4}$ film of thickness $d = 50\,\mathrm{nm}$

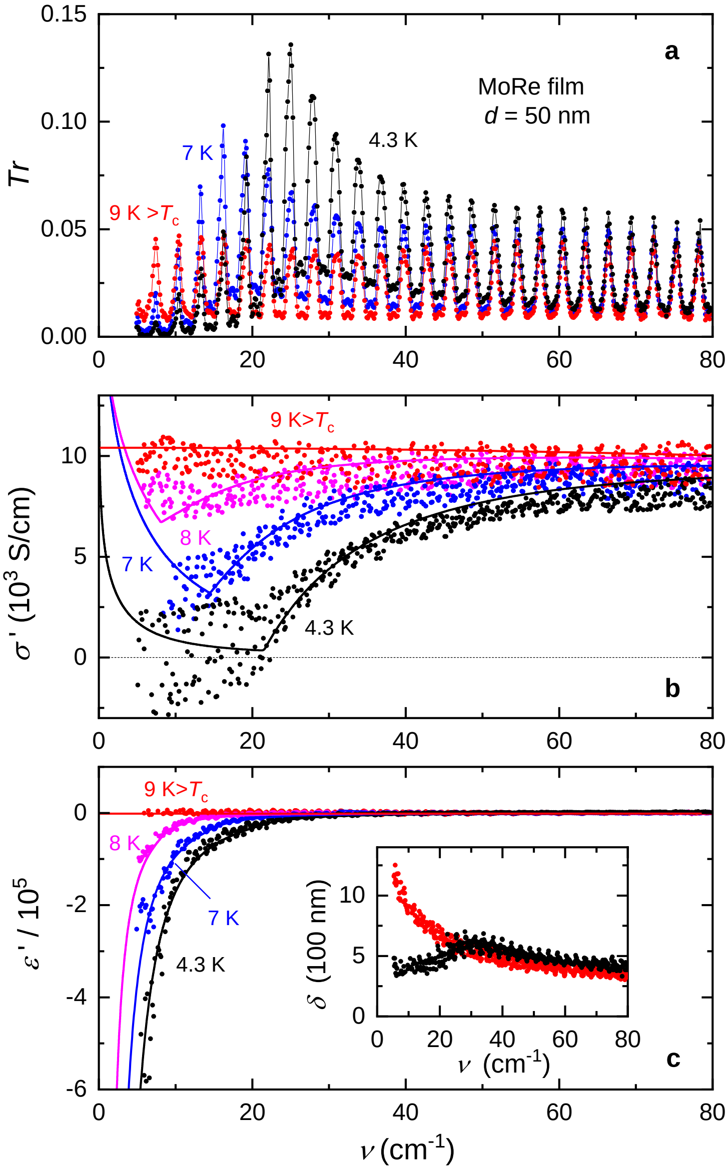

Figure S1. Spectra of transmission coefficient **a**, AC conductivity **b** and dielectric permittivity **c** of $Mo_{0.6}Re_{0.4}$ film 50 nm thick on silicon substrate, measured at different temperatures above and below T_c. Oscillations in the spectra in panel **a** are due to interference of radiation within plane-parallel Si substrate (Fabry-Perot effect). Inset in panel **c** represents penetration depth $\delta = (2\pi k\nu)^{-1}$ above and below T_c (k is extinction coefficient); vertical bar marks SC energy gap. Lines in panels **b** and **c** are least-square fits with BCS expressions from the paper by [2].

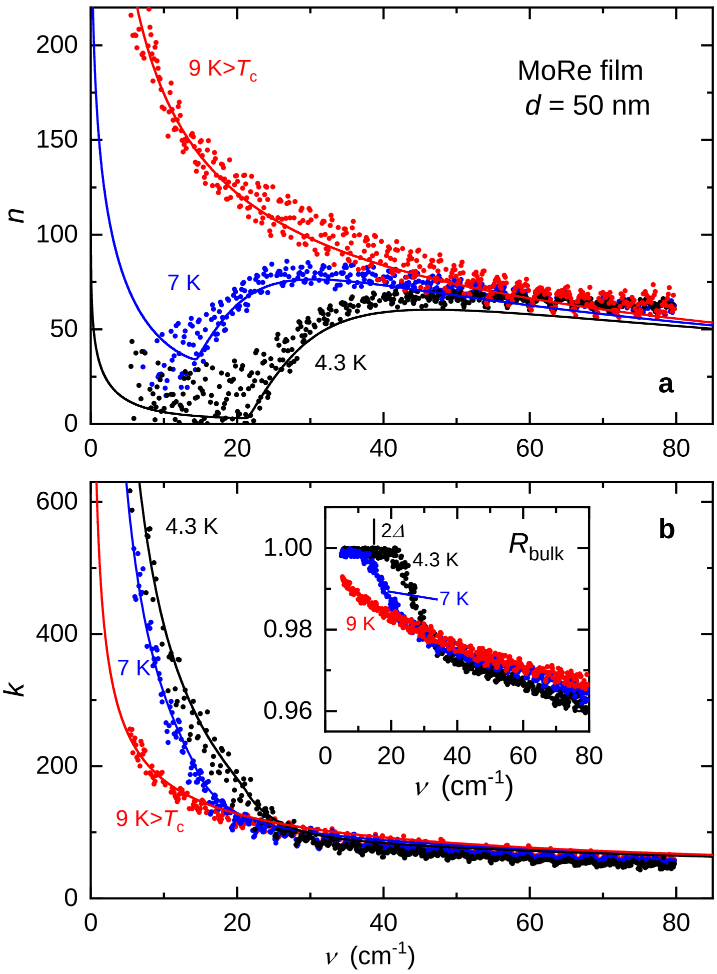

Figure S2. Spectra of refraction **a** and extinction **b** coefficients of $Mo_{0.6}Re_{0.4}$ film 50 nm thick on silicon substrate measured at temperatures above and below T_c. Lines are least-square fits with BCS expressions from the paper by [2]; lines corresponding to normal state T=9 K spectra show fits according to the low-frequency limit ($\nu \ll \gamma$) of the Drude conductivity model [1, 3], $n \approx k \approx (\sigma/\nu)^{0.5}$. Inset: corresponding bulk reflection coefficient spectra calculated according to $R_{bulk} = [(n-1)^2 + k^2]/[(n+1)^2 + k^2]$; vertical bar marks SC energy gap.

Analysis of calculated transmission spectra for the Fabry-Perot resonator made of two $Mo_{0.6}Re_{0.4}$ films

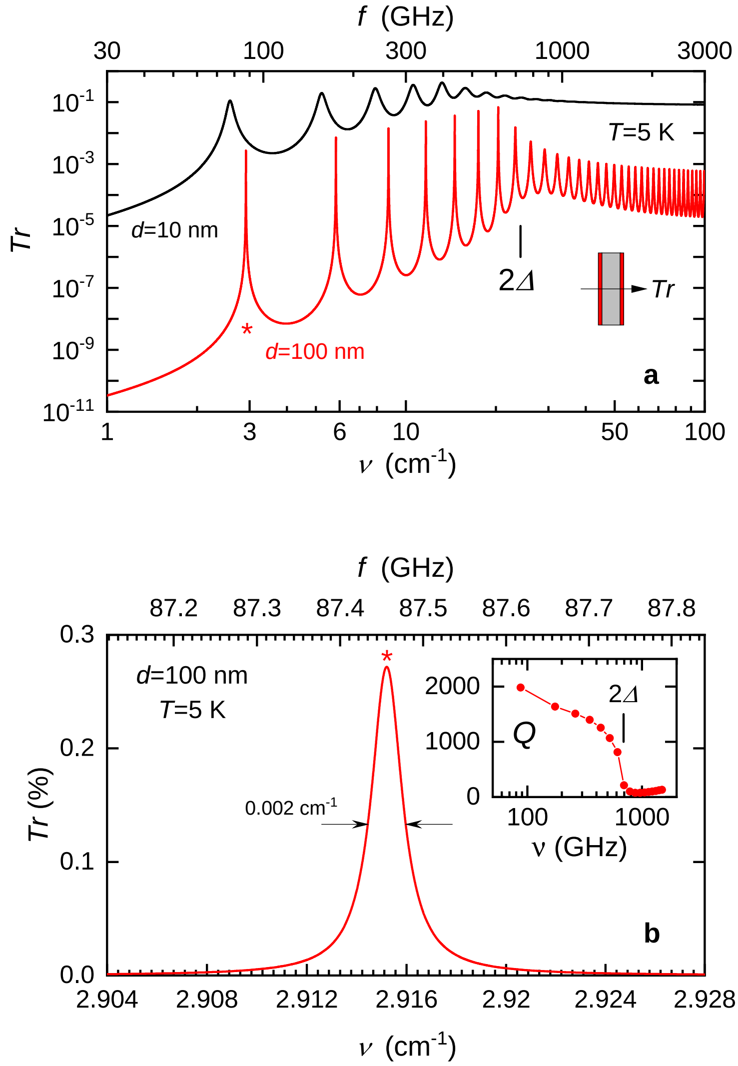

Figure S3. **a** Calculated spectra of transmission coefficient of a Fabry-Perot resonator formed by highly insulating silicon substrate (0.5 mm thick) with two superconducting $Mo_{0.6}Re_{0.4}$ films (10 nm and 100 nm thick) deposited on both faces. Electrodynamic parameters of the films are taken from fits of terahertz conductivity and permittivity spectra with BSC expressions from the paper by [2]. **b** Resonance observed around 2.9 cm^{-1} (marked by star in panels **a** and **b**) and frequency dependence of its Q-factor shown in the inset. Such narrow resonances can be used in designing advanced elements of modern optoelectronics, e.g., in frequency locking systems. Temperature T=5 K. Vertical bars mark superconducting energy gap.

References

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